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MULTIPLE BEAM FORMING NETWORKS

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ABSTRACT

Lossless beam forming networks using phase shifters and quadrature couplers are described. A technique for using the Butler network with randomly spaced arrays is discussed. A synthesis procedure for networks with N-element arrays is described. A method of using these networks with planar arrays is discussed. A least mean square algorithm for the determination of a beam forming newtork for an arbitrary array and arbitrary beam directions is developed. Computed pattern data for two thinned random planar arrays is included.

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I. INTRODUCTION

Beam forming networks are generally considered to be circuits which couple two or more antennas to one or more input/output terminals. The terminals are considered input terminals if the antennas are transmitting antennas and output terminals if the antennas are receiving antennas. A power divider network which couples the antenna elements of a linear array to a single terminal in such a fashion as to result in a linear phase taper across the face of the array is one form of a beam forming network. With this network, the array produces a beam in a direction determined by the array element spacing and the slope of the phase taper. Similarly, the hybrid which is connected to two elements to form the monopulse sum and difference patterns is a beam forming network.

Beam forming networks can be linear or nonlinear, reciprocal or nonreciprocal, passive or active, lossy or lossless, matched or not matched. They can be used to couple many antenna elements to a single port or a few antenna elements to many ports. This technical note is concerned with a single type of beam forming network. The network is linear, reciprocal, passive, lossless, matched and connects N antenna elements to N ports to form N antenna beams. The network discussed utilizes two circuit elements, the 3-dB quadrature coupler and the fixed phase shift.

The discussion will confine itself to the use of the individual antennas as elements of an array. Initially, some restrictions will be placed on the antenna array configuration. These restrictions will be removed as the discussion progresses. First, the Butler[1] Matrix is briefly discussed. Next the Nolan[2] Matrix is described. Methods for simplifying the Nolan Matrix are discussed, and the Butler Matrix is shown to be a special case of the Nolan Matrix. Finally, an extension of the Nolan Matrix to completely arbitrary array configurations is described. Computed radiation pattern data are presented for some arbitrary arrays.

II. TWO SIMPLE BEAM FORMING NETWORKS

The simplest form of the N terminal by N antenna beam forming network is shown in Figure 1. This simply connects each antenna element to each input/output port, and each beam formed is the beam of a single element. This type of network may appear trivial but in many situations it can be the best solution to a design problem. The beams formed are completely independent, the array configuration is arbitrary and not relevant to the beams, and failure of a single antenna has an effect on only one beam.

Many design problems, however, require that the beams be narrow and that the aperture size of the antenna array be small; two requirements that are in conflict with one another. For this situation, it is desirable to form each beam using as much of the available aperture space as is possible. Each I/O port should couple to more than one of the antenna elements in the array.

Figure 2 illustrates a method for doing this with a two-element array. The coupler convention utilized is indicated to the right of the figure. Note that an input to port 1 results in antenna 2 lagging antenna 1 by 90° in phase causing the beam formed to squint to the right in the figure while an input to port 2 causes the beam to squint to the left. The amount of beam squint and the width of the beams formed is primarily dependent on the distance between the antennas. The beam forming network does not require any specific antenna element spacing; two beams will be formed regardless of the spacing. If the spacing is large enough, the radiation patterns for each of the beams will include grating lobes. If the antenna requirement is such that it is desired to point the two beams formed at a narrow field of the view and the pattern structure outside this field of view is of no consequence, then the spacing can be increased to the point where either the grating lobes begin to illuminate the field of view or the width of the two formed beams reaches a desired minimum. Such a requirement might apply for a space satellite antenna illuminating the disc of the earth, for example.

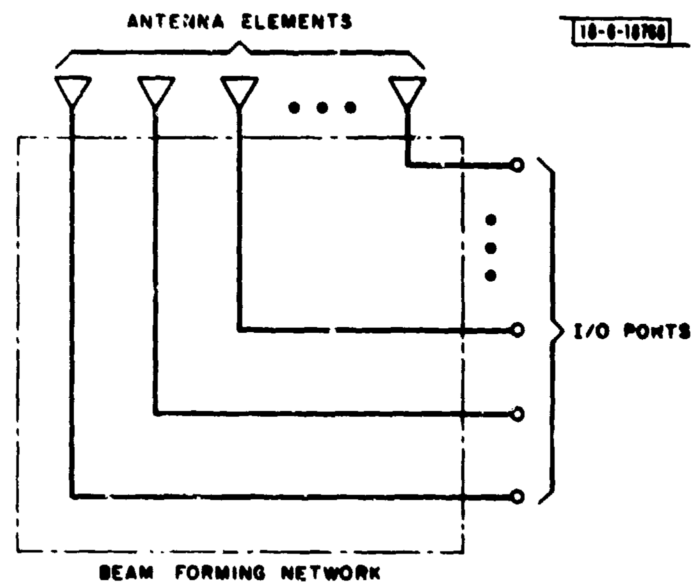


Fig. 1. N x N beamforming network-simplest form.

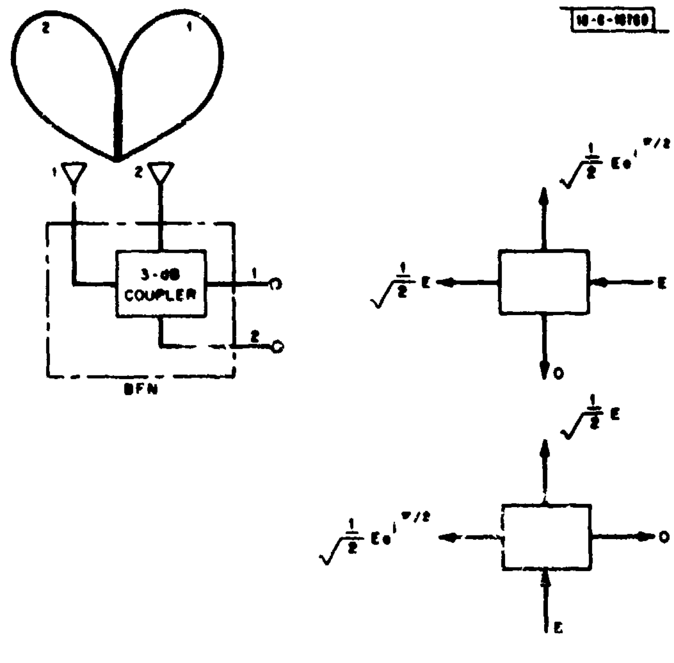


Fig. 2. Two element, two beam array.

Generally, for best performance of the network shown in Figure 2, the antenna element radiation patterns should be substantially the same, and the elements should be pointed in the same direction. Obviously, the two-element array is linear. These restrictions are imposed on the arrays which will be discussed in the next section.

III. THE BUTLER BEAM FORMING NETWORK

Butler Beam Forming Networks have been described extensively in the literature[1,3,4]. This discussion will summarize some of these descriptions with the object of extending the beam forming network concept to a class of irregularly spaced linear arrays.

Begin with the array restrictions indicated in the previous section and further that the elements in the linear array are equally spaced and that their number is equal to an integral power of two. Since there are an even number of elements, hence beams, it is reasonable to have the beams equally distributed about the array boresight; N/2 beams to the left of boresight and N/2 beams to the right of boresight. (Boresight is defined as the direction perpendicular to the line of the array.) Also, since the elements are equally spaced, a linear phase taper, which determines a beam direction, is formed by equal phase increments between elements such that the phase of the k^{th} element for the n^{th} beam can be specified as:

$$\phi_{n,k} = \phi_{n,1} + \frac{(2n-1)(k-1)}{N}\pi \qquad (1)$$

where $\phi_{n,1}$ is determined by the network chosen. Table I shows a listing of the second term of (1) in multiples of π for an eight-element array. The synthesis of the network required to form these phase tapers is described elsewhere[1] and is not presented here. The resultant network is shown in Figure 3.

Figure 3 indicates a linear, equally spaced array of antenna elements. The element spacing, however, can be made irregular to some extent[5]. Figure 4 shows a plot of the phase taper vs. element position for a typical beam. The plot shows the phase taper for the eight equally spaced elements of Figure 3a in solid line and the taper required for additional elements if the array were to be extended without disturbing the element spacing or the beam direction in dotted line. The dashed line taper would be required if the elements -4 through 0 and 9 through 13 were reversed 180° in phase. This can generally be accomplished by physically rotating the antenna elements 180°. It can be seen from the curve that since the phase taper can be drawn as a periodic

TABLE I

RELATIVE PHASE TERMS - EIGHT ELEMENT LINEAR ARRAY

$$\frac{\beta_{n,k}}{\pi} = \frac{(2n-1)(k-1)}{N}$$

k→ n ↓	1	2	3	4	5	6	7	8
1	0	1/8	1/4	3/8	1/2	5/8	3/4	7/8
2	0	3/8	3/4	1-1/8	1-1/2	1-7/8	2-1/4	2-5/8
3	0	5/8	1-1/4	1-7/8	2-1/2	3-1/8	3-3/4	4-3/8
4	0	7/8	1-3/4	2-5/8	3-1/2	4-3/8	5-1/4	6-1/8
5	0	1-1/8	2-1/4	3-3/8	4-1/2	5-5/8	6-3/4	7-7/8
6	0	1-3/8	2-3/4	4-1/3	5-1/2	6-7/8	8-1/4	9-5/8
7	0	1-5/8	3-1/4	4-7/8	6-1/2	8-1/8	9-3/4	11-3/8
8	0	1-7/8	3-3/4	5-5/8	7-1/2	9-3/8	11-1/4	13-1/8

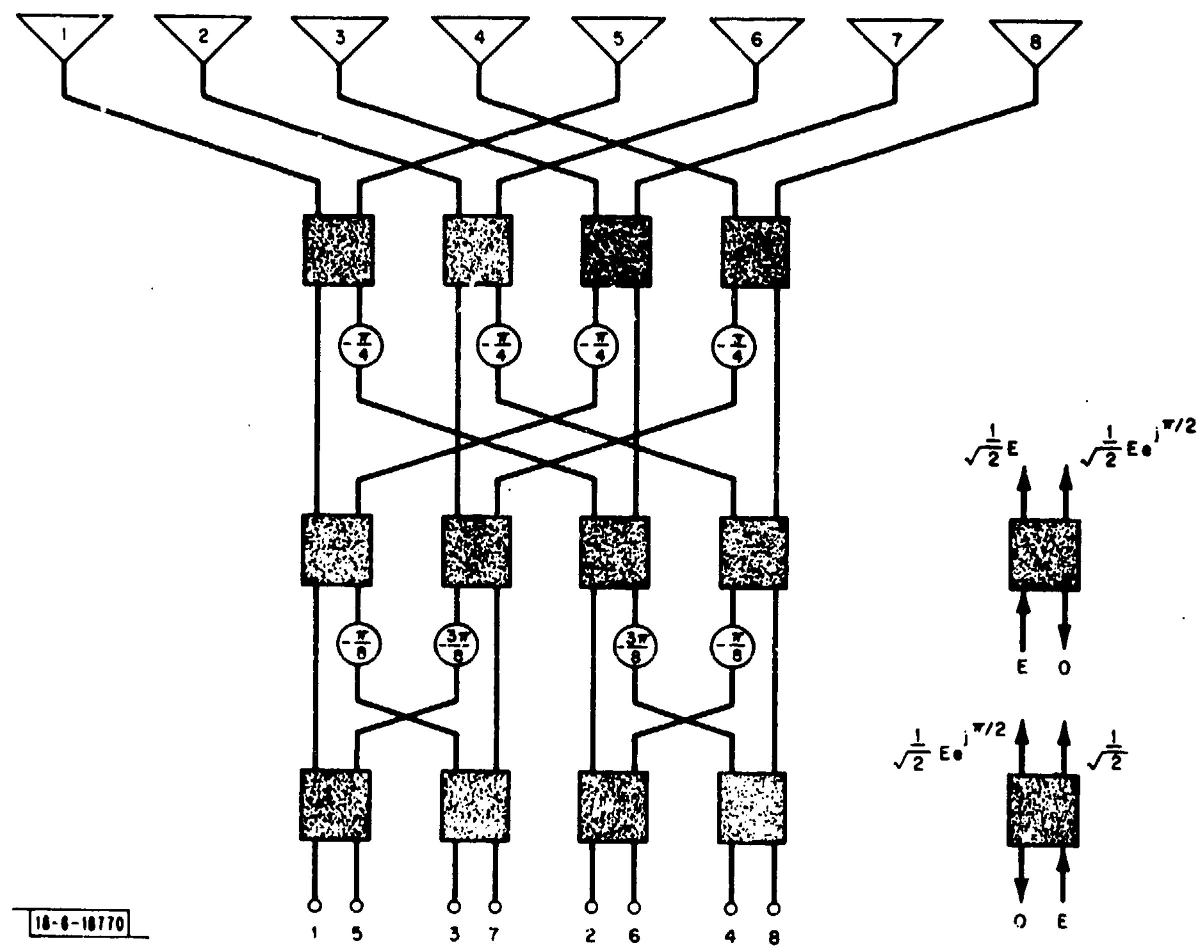


Fig. 3. Eight-element Butler BFN.

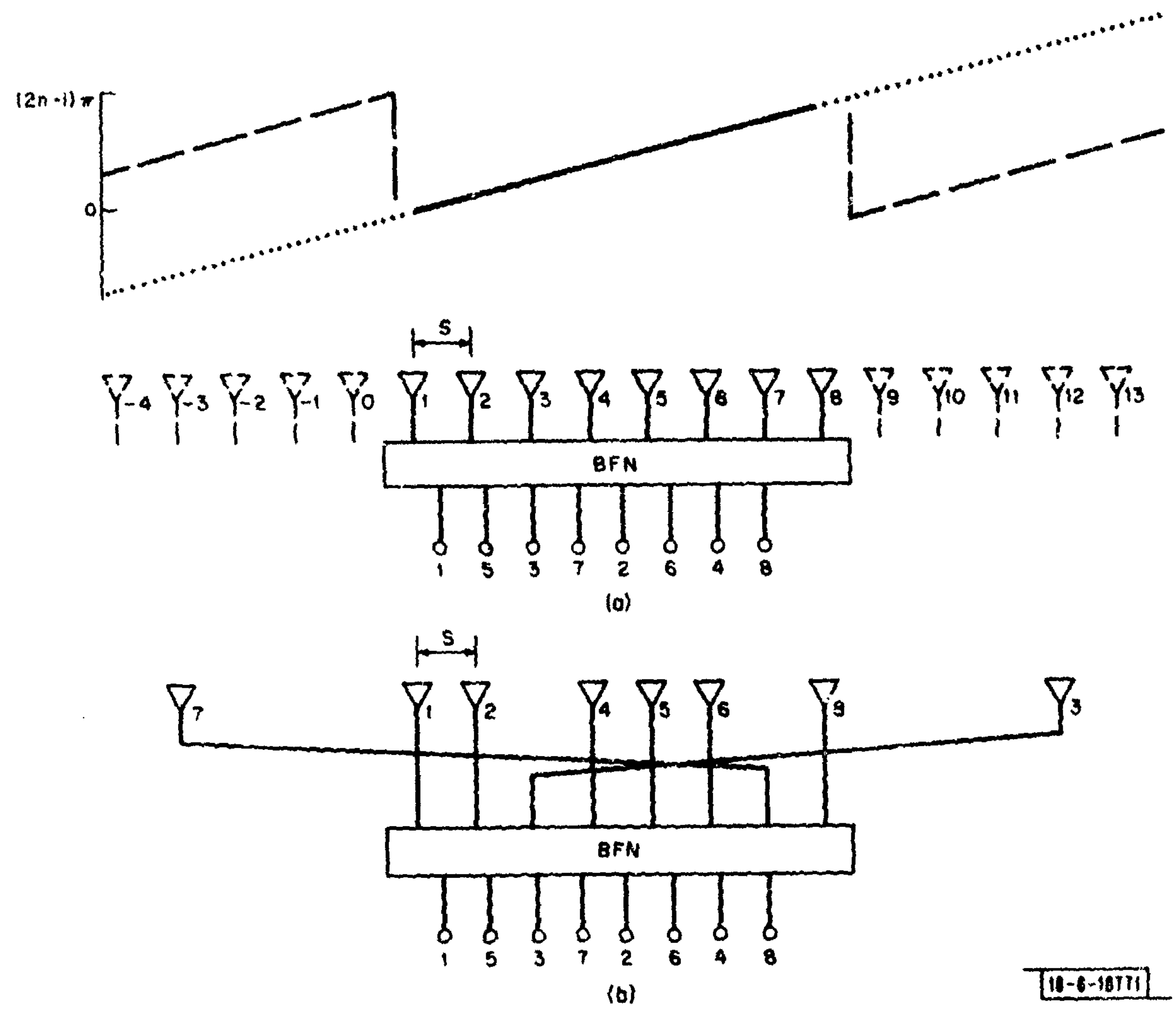


Fig. 4. Irregularly spaced Butler array.

function, elements within the bounds of the original array can be moved to positions outside these bounds where the required phase, including the 180° reversal, is no different from that at the original element position. This is done with elements 3 and 7 in Figure 3b to produce an irregularly spaced array.

The Butler Beam Forming Network can then be used for both equally spaced linear arrays and a class of randomly spaced linear arrays. It is, however, restricted to arrays which have the number of elements equal to an integral power of 2. Nolan[2] removed this restriction.

IV. THE NOLAN SYNTHESIS

The Butler Beam Forming Network is restricted to arrays which have the number of elements equal to an integral power of 2. The use of a Butler network with arrays with different numbers of elements usually requires the introduction of some loss into the network or the use of couplers with more than four ports. In the latter case, if six-port couplers were utilized rather than four-port couplers, the arrays which have the number of elements equal to an integral power of three could be utilized. For the former case, an N element array could be used with an M element Butler network (M greater than or equal to N) with the unused outputs of the network terminated. This results in a reduction in gain by a factor $(\frac{N}{M})^2$.

It is possible, however, to determine a beam forming network for any arbitrary number of array elements provided that the network transfer function, A, has specific properties. In general, A can be written as an NxN matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdot & \cdot & \cdot & \cdot & a_{1N} \\ a_{21} & a_{22} & \cdot & \cdot & \cdot & \cdot & a_{2N} \\ \cdot & \cdot & & & & & \\ \cdot & & \cdot & & & & \\ \cdot & & & \cdot & & & \\ \cdot & & & & \cdot & & \\ \cdot & & & & & \cdot & \\ a_{N1} & a_{N2} & \cdot & \cdot & \cdot & \cdot & a_{NN} \end{bmatrix}$$

where $a_{n,k}$ is the transfer coefficient between the n^{th} beam port and the k^{th} array element and

$$a_{n,k} = b_{n,k}\, e^{j\theta nk} \quad , \quad \text{where } b_{n,k} \text{ is a real number.} \tag{3}$$

If the beam forming network is to be lossless, linear, reciprocal, and matched, the matrix A must have the following properties:

1. The sum of the squares of the magnitudes of the terms in any row or column is unity.
2. The sum of the products of the terms in any row or column with the conjugate of the corresponding terms in any other row or column is zero.

These properties are an expression of conservation of energy, i.e., in a lossless matched system, the power in equals the power out. The first of these simply states that the power radiated by each array element for a given beam is equal to the power input to the corresponding beam port. The second states that if a received signal induces voltages in the array elements which are equal to the conjugate of the transfer coefficients for a given beam, then all of the received power will be coupled to that beam port, and no power will couple to any other port. Because the network is reciprocal, it may be reversed, the beam ports and element ports interchanged, without affecting its performance; hence the statements apply to both rows and columns of the transfer matrix.

These are the properties of a unitary matrix and can be stated mathematically in matrix notation as

$$A^T A = I \tag{4}$$

where I is the identity matrix defined by

$$i_{n,k} = \begin{cases} 1 & n=k \\ 0 & n \neq k \end{cases}$$

and $(\ )^T$ represents the conjugate transpose of the matrix.

One of the properties of a unitary matrix is that it can be factored into the product of elementary unitary matrices. A particular form of elementary unitary matrix which is of interest because it represents a simple r.f. circuit is

$$E_1 = \begin{bmatrix} \cos\theta\, e^{j(\phi + \frac{\pi}{2})} & \sin\theta\, e^{-j\frac{\pi}{2}} & 0 & 0 \dots 0 \\ \sin\theta\, e^{j(\phi + \frac{\pi}{2})} & \cos\theta\, e^{+j\frac{\pi}{2}} & 0 & 0 \dots 0 \\ 0 & 0 & 1 & 0 \dots 0 \\ 0 & 0 & 0 & 1 \dots 0 \\ \vdots & \vdots & \vdots & \\ 0 & 0 & 0 & 0 \dots 1 \end{bmatrix} \tag{5}$$

The circuit which has E_1 as a transfer function is shown in Figure 5.

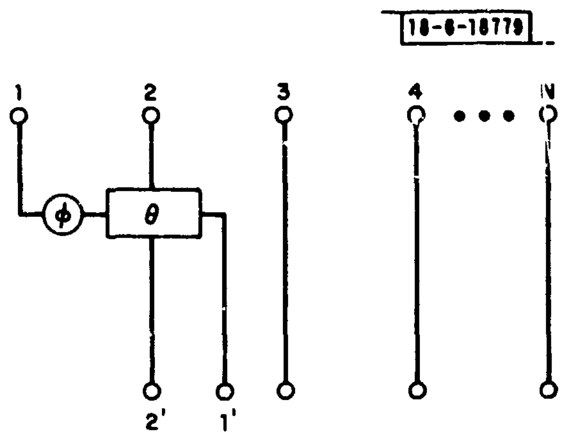


Fig. 5. Circuit representation of N x N elementary unitary matrix.

Where the element θ is a directional coupler with the properties as shown in Figure 6.

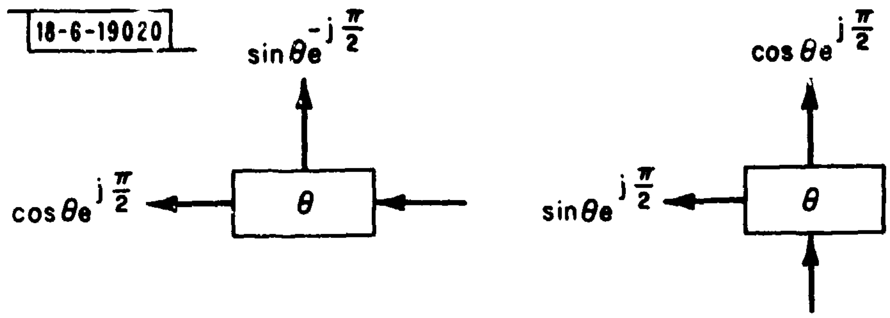


Fig. 6. Directional coupler transfer properties.

This elementary circuit provides a means for determining a beam forming network for the general N element array. The transfer function of the array is specified by (2) and (3) and can be drawn in circuit form as shown in Figure 7.

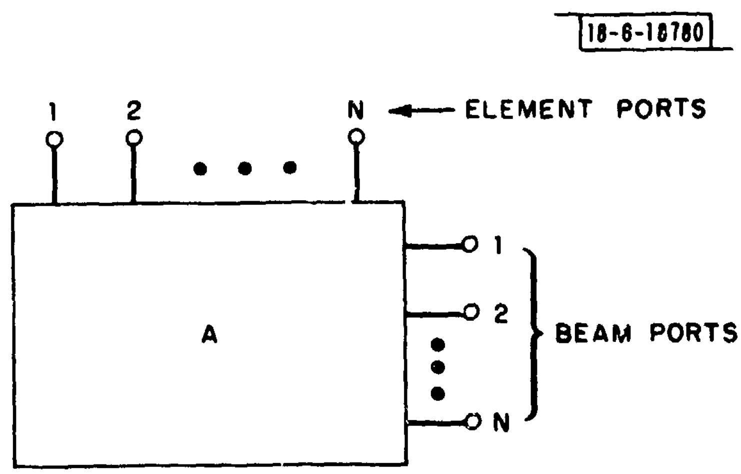


Fig. 7. Beam forming network before reduction.

The circuit A can be also configured, using the circuit in Figure 5, as shown in Figure 8:

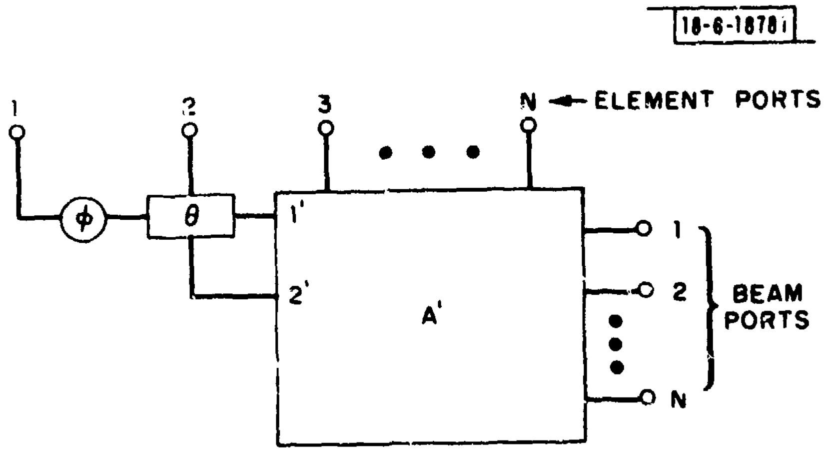


Fig. 8. Beam forming network - first Nolan reduction.

Comparing Figures 7 and 8, and making use of the relations in Figure 6, it can be seen by inspection that

$$a_{n,1} = a'_{n,1} \cos\theta \, e^{j(\phi + \frac{\pi}{2})} + a'_{n,2} \sin\theta \, e^{j(\phi + \frac{\pi}{2})} \qquad (6)$$

$$a_{n,2} = a'_{n,1} \sin\theta \, e^{-j\frac{\pi}{2}} + a'_{n,2} \cos\theta \, e^{j\frac{\pi}{2}} \qquad (7)$$

or, in matrix notation

$$A'E_1 = A$$

The circuit A' can then be reduced in the same manner as was the circuit A by operating on terminals 1' and 3 with a similar coupler, such that the circuit in Figure 9 is obtained.

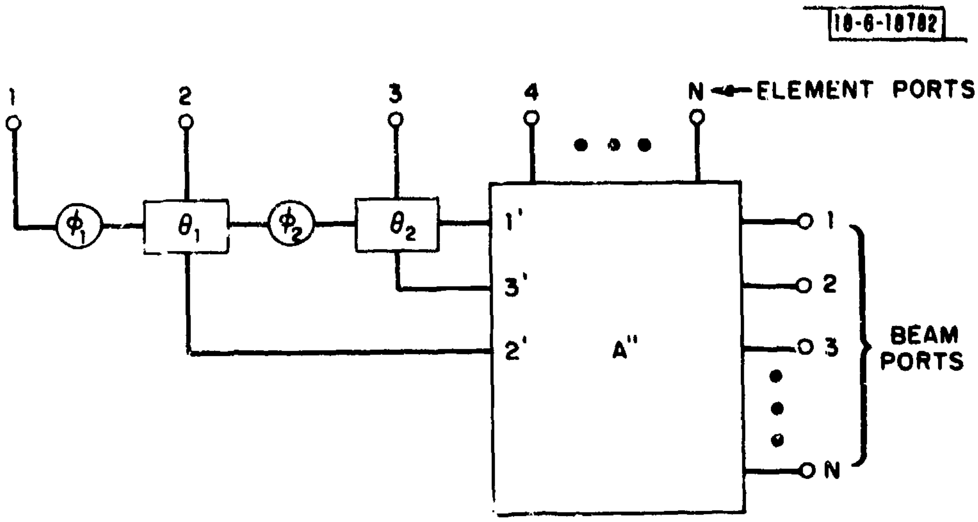


Fig. 9. Beam forming network - second Nolan reduction.

and

$$(A''E_2)E_1 = A'E_1 = A$$

Since this second coupler operates on ports 1' and 3, its transfer function is expressed as:

$$E_2 = \begin{bmatrix} \cos\theta_2 \, e^{j(\phi_2 + \frac{\pi}{2})} & 0 & \sin\theta_2 \, e^{-j\frac{\pi}{2}} & 0 \ldots 0 \\ 0 & 1 & & 0 \ldots 0 \\ \sin\theta_2 \, e^{j(\theta_2 + \frac{\pi}{2})} & 0 & \cos\phi_2 \, e^{j\frac{\pi}{2}} & 0 \ldots 0 \\ 0 & 0 & & 1 \ldots 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & & 0 \ldots 1 \end{bmatrix}$$

Continuing the same process for N-1 couplers results in the circuit in Figure 10.

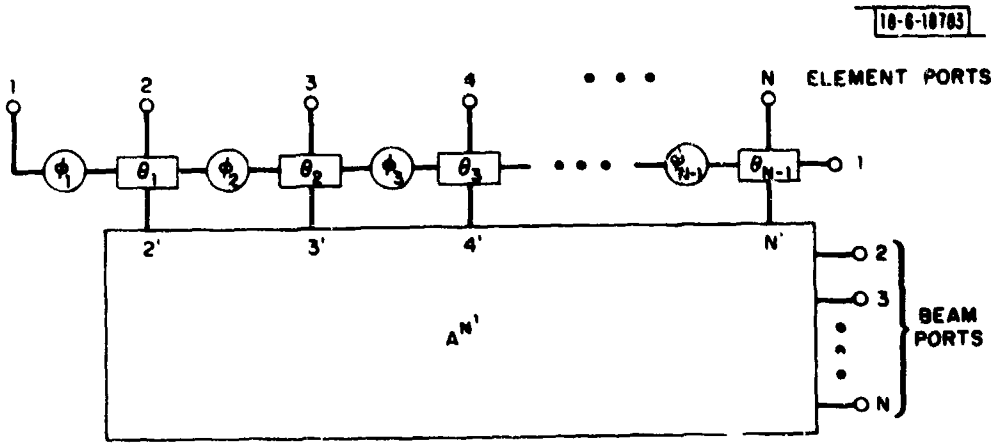


Fig. 10. Beam forming network - N-1st Nolan reduction.

where the transfer function $A^{N'}$ is an an N-1 matrix, a matrix of order less than the original matrix A. The circuit $A^{N'}$ can then be reduced in the same manner as was A, starting with the 2' and 3' ports and so on, until the circuit is completely reduced to the form shown in Figure 11.

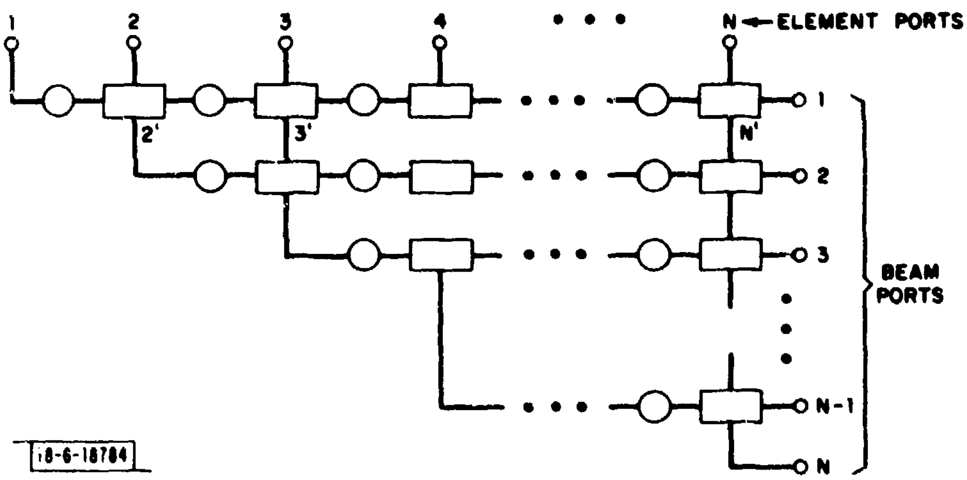


Fig. 11. Beam forming network - completely reduced.

It remains now to determine the values of θ and ϕ throughout the circuit, to complete the synthesis. The two preceding circuits show that there is no coupling between beam port 1 and the port 2' of $A^{N'}$ and similarly of A' because, as shown in Figure 6, an input from the right of any coupler is not coupled to its bottom port. This means that the matrix element $a'_{1,2}$ is equal to zero. This allows Equations (6) and (7) to be solved for θ_1 and ϕ_1 and for $a'_{1,1}$. Similarly, $a''_{1,3}$ is equal to zero and θ_2 and ϕ_2 can be determined. This procedure can be continued, and the circuit synthesis then is thus one requiring successive solutions to equations of the form of (6) and (7). Using matrix notation, this method can be succinctly expressed. Since A is unitary, it can be expressed as the product of the factors

$$A = E_{N(N-1)/2}\, E_{N(N-1)/2\,-1} \cdots E_3 E_2 E_1$$

where E_N is an elementary matrix of the general form

$$E = \begin{bmatrix} \cos\theta\, e^{j(\phi + \frac{\pi}{2})} & 0 & \sin\theta\, e^{-j\frac{\pi}{2}} & \cdots & 0 \\ 0 & 1 & 0 & & 0 \\ \sin\theta\, e^{j(\phi + \frac{\pi}{2})} & 0 & \cos\theta\, e^{j\frac{\pi}{2}} & & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & & 0 \quad 0 & & 1 \end{bmatrix}$$

Then successive post multiplications of A by E_1^T, E_2^T, etc., will eventually reduce A to an identity matrix. The procedure is to choose the θ_n, ϕ_n parameters in E_1 through E_{N-1} so as to force successive terms in the upper row of the product matrix to zero until all of the terms in the row are zero except the first term. Because the matrix is unitary, the first term in the row will be unity, and all other terms in the first column will be zero. This reduces the N x N transfer matrix to an N-1 x N-1 transfer matrix. The second row is reduced in a similar way by proper choice of the next N-2 values of E_n, etc., until all values of θ_n and ϕ_n are determined.

The circuit synthesized by this method uses fixed phase shifts and quadrature couplers which, in general, are not 3 dB couplers. These can be made using 3 dB couplers using the circuit in Figure 12, however.

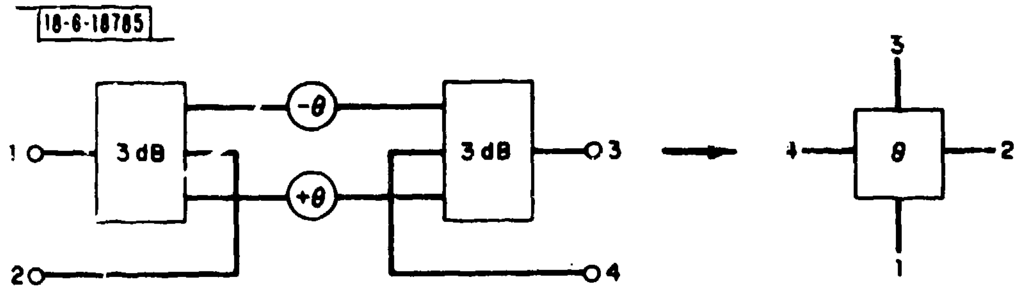


Fig. 12. Directional coupler circuit using 3 dB couplers.

The transfer function is then:

	3	4
1	$\cos\theta\ e^{j\frac{\pi}{2}}$	$\sin\theta\ e^{j\frac{\pi}{2}}$
2	$\sin\theta\ e^{-j\frac{\pi}{2}}$	$\cos\theta\ e^{j\frac{\pi}{2}}$

as is required for the coupler characteristic chosen for the synthesis.

This synthesis procedure is applicable to an array of any number of elements in any desired configuration. The only restriction is that the desired transfer matrix be unitary. Figure 13 shows the circuit for a 5 element array synthesized by this procedure where the terms of the transfer matrix take the form

$$a_{n,k} = \frac{1}{\sqrt{5}}\ e^{j(n-1)(k-1)\frac{2\pi}{5}}$$

for the n^{th} beam port and the k^{th} array element. This network can be used for a five element linear array with equally spaced elements and uniform amplitude aperture distribution.

V. A NETWORK SIMPLIFICATION

The form of the beam port/element port transfer coefficients for the network of Figure 13 is generally applicable to all linear equally spaced arrays and, as will be described in a later section, to some planar arrays as well. For uniform illumination, the desired network transfer coefficients will usually take the form

$$a_{n,k} = \frac{1}{\sqrt{N}} \; e^{j(n-1)(k-1)\frac{2\pi}{N}} \quad . \tag{8}$$

This can be simplified somewhat by noting that the N-(k-2)st terms in each row have the values

$$\begin{aligned} a_{n,[N-(k-2)]} &= \frac{1}{\sqrt{N}} \, e^{j(n-1)[N-(k-1)]\frac{2\pi}{N}} \\ &= \frac{1}{\sqrt{N}} \, e^{-j(n-1)(k-1)\frac{2\pi}{N}} \, e^{j(n-1)2\pi} \end{aligned}$$

and since $e^{j(n-1)2\pi} = 1$

$$a_{n,[n-(k-2)]} = \frac{1}{\sqrt{N}} \, e^{-j(n-1)(k-1)\frac{2\pi}{N}}$$

which is the conjugate of the term $a_{n,k}$. A similar argument for the N-(n-2)st terms in each column can also be made. This suggests a possible simplification of the network to be synthesized.

Consider a 3 dB-coupler circuit with inputs $\sqrt{2}\cos\alpha$ and $\sqrt{2}\sin\alpha$. As indicated in Figure 14, the outputs will take the form $e^{j\alpha}$ and $e^{-j\alpha}$.

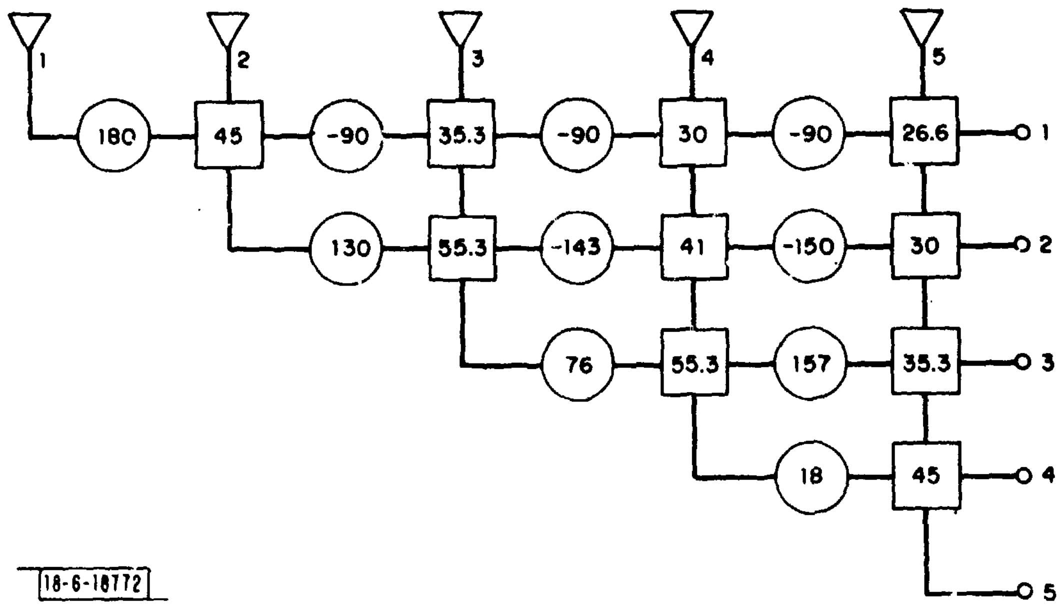


Fig. 13. Five-element array BFN.

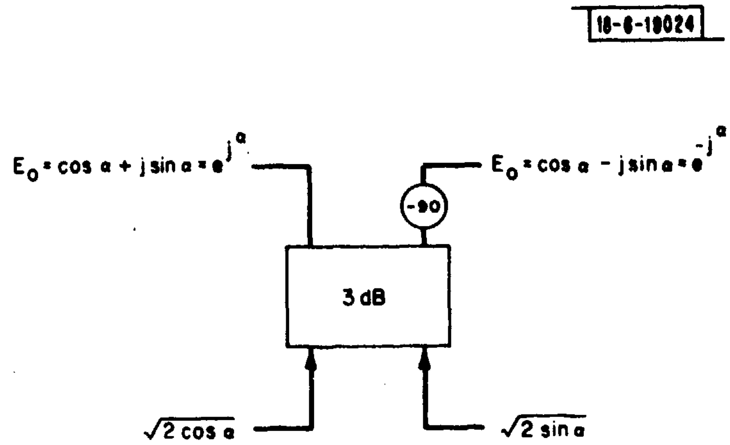


Fig. 14. 3 dB coupler circuit transfer properties.

Using this property, the beam forming network can be represented as in Figure 15 where the transfer function of the network B takes the form (when N is odd):

$$B = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & \sqrt{2} & \sqrt{2} & \cdots & 0 & 0 \\ 1 & \sqrt{2}\cos\frac{2\pi}{N} & \sqrt{2}\cos\frac{4\pi}{N} & \cdots & \sqrt{2}\sin\frac{4\pi}{N} & \sqrt{2}\sin\frac{2\pi}{N} \\ 1 & \sqrt{2}\cos\frac{4\pi}{N} & \sqrt{2}\cos\frac{4\pi}{N} & \cdots & \sqrt{2}\sin\frac{8\pi}{N} & \sqrt{2}\sin\frac{4\pi}{N} \\ & \vdots & . & & \vdots & \\ 1 & \sqrt{2}\cos(-\frac{4\pi}{N}) & \sqrt{2}\cos(-\frac{8\pi}{N}) & \cdot\,\cdot & \sqrt{2}\sin(-\frac{8\pi}{N}) & \sqrt{2}\sin(-\frac{4\pi}{N}) \\ 1 & \sqrt{2}\cos(-\frac{2\pi}{N}) & \sqrt{2}\cos(-\frac{4\pi}{N}) & \cdot\,\cdot & \sqrt{2}\sin(-\frac{4\pi}{N}) & \sqrt{2}\sin(-\frac{2\pi}{N}) \end{bmatrix}$$

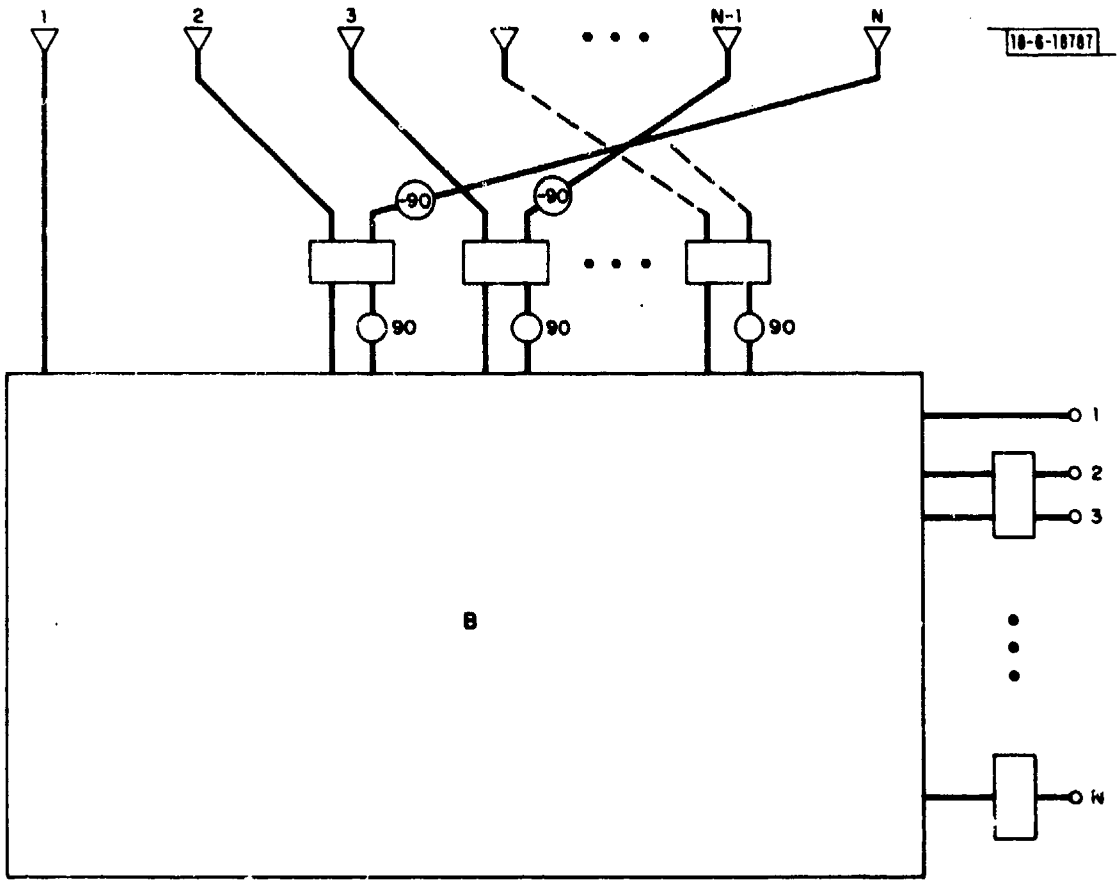


Fig. 15. Beam forming network - first reduction.

Figure 15 shows that the matrix B can be treated as two separate matrices B_1 and B_2 where B_2 is comprised of the first $\frac{N+1}{2}$ columns of B, and B_2 is comprised of the remaining columns.

$$B_1 = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & \sqrt{2} & \sqrt{2} & \cdots & \sqrt{2} \\ 1 & \sqrt{2}\cos\frac{2\pi}{N} & \sqrt{2}\cos\frac{4\pi}{N} & \cdots & \sqrt{2}\cos(\frac{N-1}{2})\frac{2\pi}{N} \\ 1 & \sqrt{2}\cos\frac{4\pi}{N} & \sqrt{2}\cos\frac{8\pi}{N} & \cdots & \sqrt{2}\cos(N-1)\frac{2\pi}{N} \\ & \vdots & & & \\ 1 & \sqrt{2}\cos -\frac{4\pi}{N} & \sqrt{2}\cos -\frac{8\pi}{N} & \cdots & \sqrt{2}\cos -(N-1)\frac{2\pi}{N} \\ 1 & \sqrt{2}\cos -\frac{2\pi}{N} & \sqrt{2}\cos -\frac{4\pi}{N} & \cdots & \sqrt{2}\cos -(\frac{N-1}{2})\frac{2\pi}{N} \end{bmatrix}$$

$$B_2 = \frac{1}{\sqrt{N}} \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \sqrt{2}\sin\frac{2\pi}{N} & \sqrt{2}\sin\frac{4\pi}{N} & \cdots & \sqrt{2}\sin(\frac{N-1}{2})\frac{2\pi}{N} \\ \sqrt{2}\sin\frac{4\pi}{N} & \sqrt{2}\sin\frac{8\pi}{N} & \cdots & \sqrt{2}\sin(N-1)\frac{2\pi}{N} \\ \vdots & & & \\ \sqrt{2}\sin -\frac{4\pi}{N} & \sqrt{2}\sin -\frac{8\pi}{N} & \cdots & \sqrt{2}\sin -(\frac{N-1}{2})\frac{2\pi}{N} \\ \sqrt{2}\sin -\frac{2\pi}{N} & \sqrt{2}\sin -\frac{4\pi}{N} & \cdots & \sqrt{2}\sin -(\frac{N-1}{2})\frac{2\pi}{N} \end{bmatrix}$$

Examination of B_1 shows that the n^{th} row is ide[illegible] to the N-(n-2)st row since $\cos\alpha = \cos(-\alpha)$. Examination of B_2 shows that the N-(n-2)st row is the negative of the n^{th} row since $\sin\alpha = -\sin(-\alpha)$. This suggests that the number of rows in each matrix might be reduced since the difference between the formation of the n^{th} beam and the (N-(n-2)st beam is simply the sign of the terms in the B_2 matrix.

A coupler with the properties shown in Figure 16 serves the purpose for this.

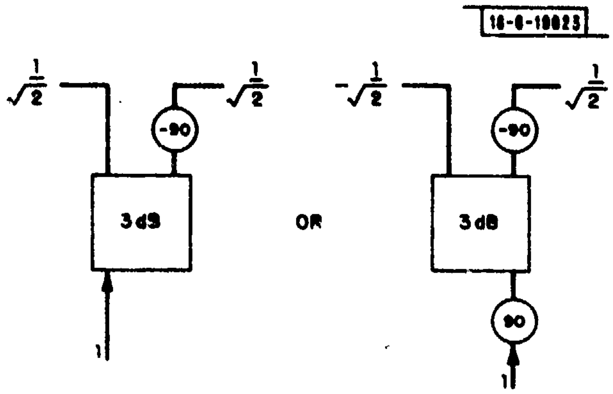


Fig. 16. 3 dB coupler circuit transfer properties.

and the network becomes that shown in Figure 17. The 90° phase shifter at the beam inputs indicated in Figure 16 is omitted in Figure 17 since this affects only the relative phase of the beam. The transfer matrices of the networks C_1 and C_2 are

$$C_1 = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & \sqrt{2} & \cdots & \sqrt{2} \\ \sqrt{2} & 2\cos\frac{2\pi}{N} & \cdots & 2\cos\left(\frac{N-1}{2}\right)\frac{2\pi}{N} \\ \vdots & & & \\ \sqrt{2} & 2\cos\left(\frac{N-1}{2}\right)\frac{2\pi}{N} & \cdots & 2\cos\left(\frac{N-1}{2}\right)^2\frac{2\pi}{N} \end{bmatrix}$$

$$C_2 = \frac{1}{\sqrt{N}} \begin{bmatrix} 2\sin\frac{2\pi}{N} & \cdots\cdots & 2\sin\left(\frac{N-1}{2}\right)\frac{2\pi}{N} \\ \vdots & & \\ 2\sin\left(\frac{N-1}{2}\right)\frac{2\pi}{N} & \cdots\cdots & 2\sin\left(\frac{N-1}{2}\right)^2\frac{2\pi}{N} \end{bmatrix}$$

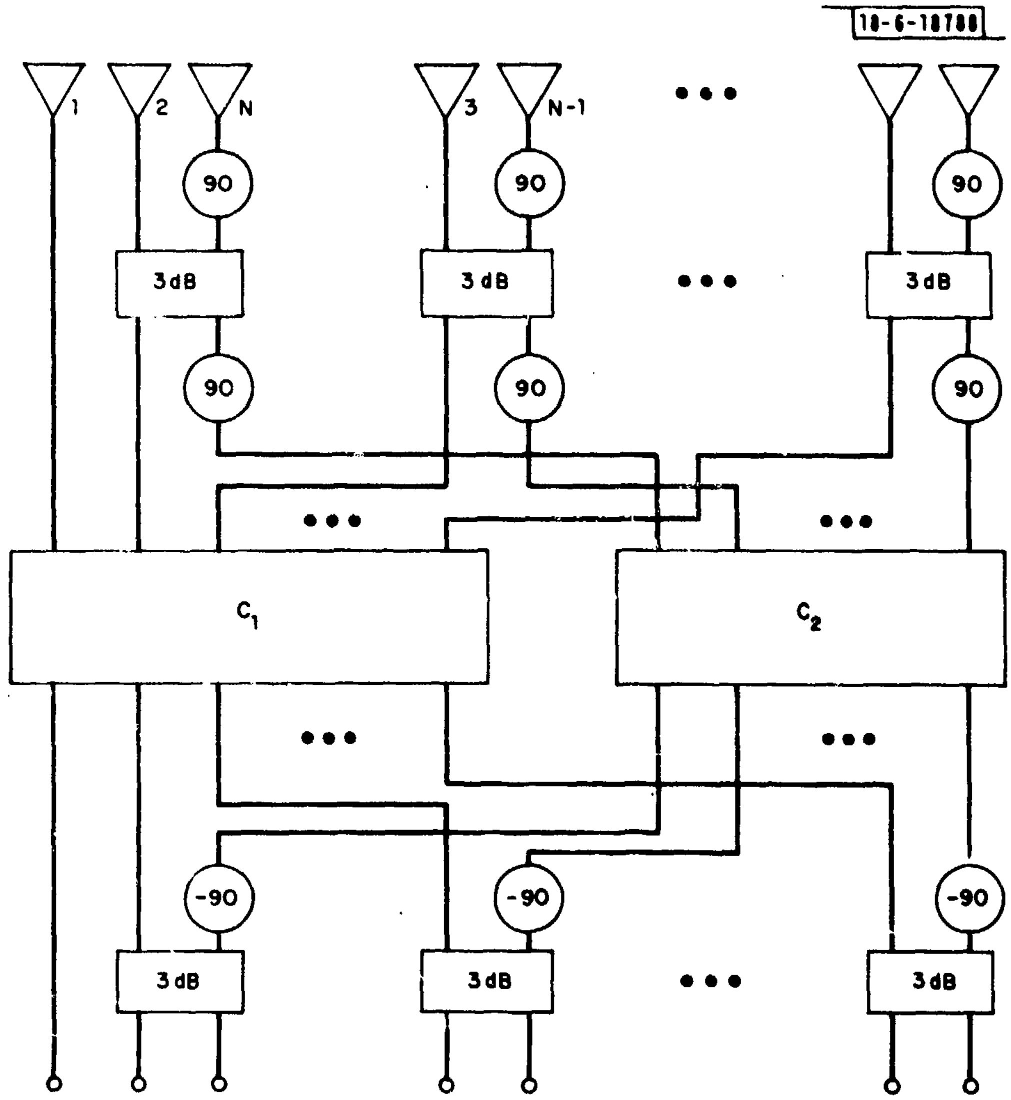


Fig. 17. Beam forming network - second reduction.

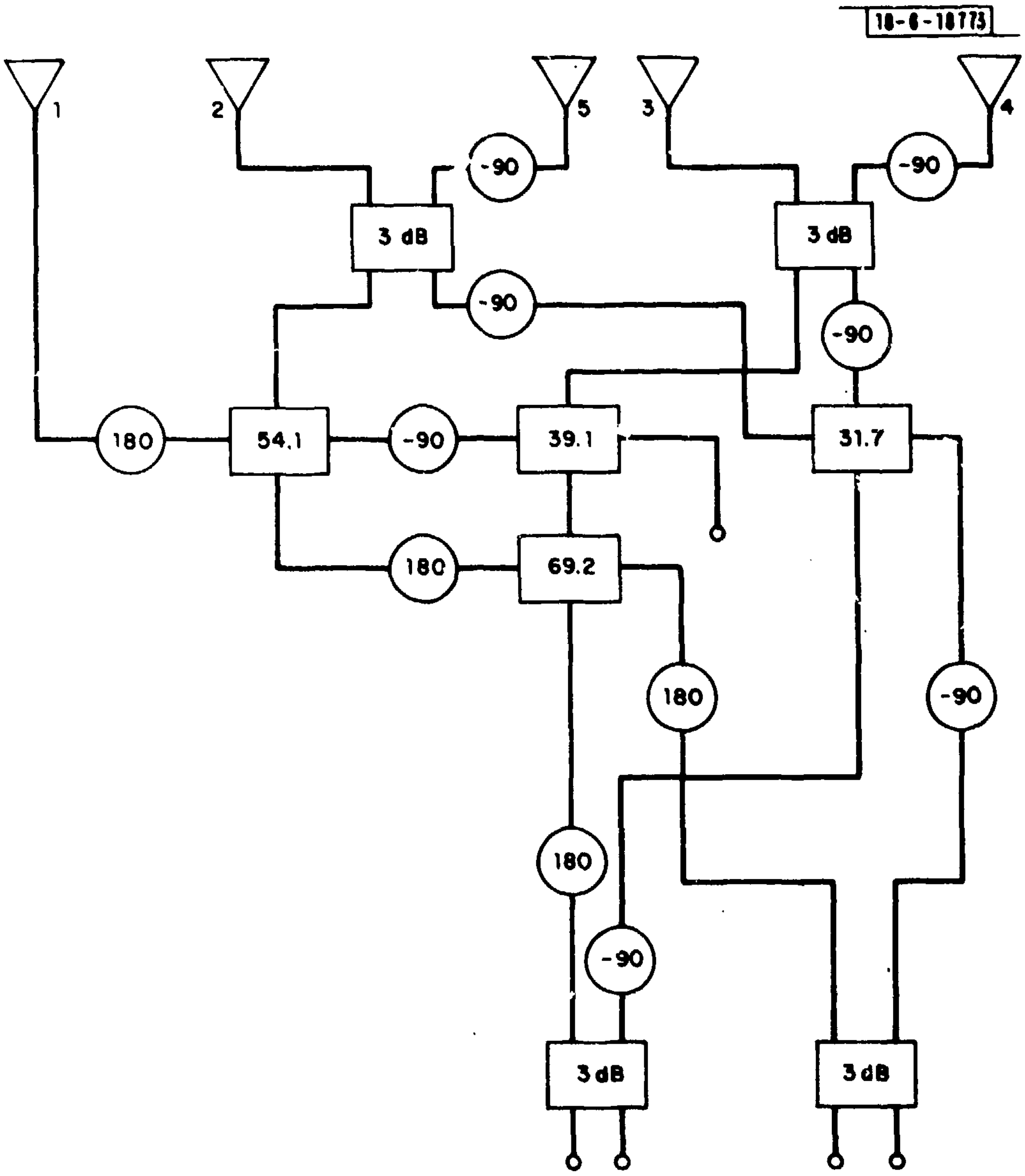


Fig. 18. Simplified 5-element BFN.

In general, the Nolan synthesis of an N x N network will require $\frac{N(N-1)}{2}$ of the θ type couplers described in the previous section or, as previously indicated, twice as many 3 dB couplers, N(N-1). The five element array in Figure 13 uses 10 of the θ couplers, and if fabricated with 3 dB couplers would require 20 of these. Examination of Figure 13 shows that this number can be reduced to 18 since the two 45° couplers are already 3 dB couplers. The preceding simplification for transfer coefficients of the form (8) where N is odd require the synthesis of two networks, one $\frac{N+1}{2} \times \frac{N+1}{2}$ and the other $\frac{N-1}{2} \times \frac{N-1}{2}$. Together these matrices would require $\frac{(N-1)(N-1)}{2 \times 2}$ of the θ type couplers and N-1 3 dB couplers or $\frac{N^2-1}{2}$ 3 dB couplers. The total number of 3 dB couplers required is reduced by a factor $\frac{N+1}{2N}$. This reduces the number required for a five-element array to twelve 3 dB couplers. The five-element array network has been synthesized using the simplification, and the result is shown in Figure 18.

Thus far, the discussion of simplification of the network has been concerned with networks for arrays which consist of an odd number of antenna elements. Because of this, the first column of the A transfer matrix was left undisturbed in the conversion to the B transfer matrix. If the number of elements and beams is even, a matrix with transfer coefficients of the form (8) is generally unsuitable. This matrix would include an element-to-element phase shift of 180°. This results in a beam, for a linear array, which points along the axis of the array in both directions. A more suitable transfer matrix for even numbered arrays is the form used for the Butler Matrix

$$a_{n,k} = \frac{1}{\sqrt{N}} e^{j(2n-1)(k-1)\frac{\pi}{N}} \quad . \tag{9}$$

This form results from adding a phase shift of $(k-1)\frac{\pi}{N}$ to the k^{th} element port of a matrix with transfer coefficients of the form (8).

Using a desired transfer matrix of the form (9) with N even, the rows and columns of the matrix having a 0-180° or ± 90° relationship may first be combined with 3 dB couplers. This reduces the transfer matrix to two $\frac{N}{2} \times \frac{N}{2}$ transfer matrices. If $\frac{N}{2}$ is odd, then each matrix may be simplified as described

for arrays with odd numbers of elements. If $\frac{N}{2}$ is even, then the matrix may be further reduced. The simplification of the eight element network is presented as an example.

Using (9), the A matrix is

$$
A = \frac{1}{\sqrt{8}} e^{j}
\begin{array}{c}
\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\
\begin{bmatrix}
0 & \frac{\pi}{8} & \frac{\pi}{4} & \frac{3\pi}{8} & \frac{\pi}{2} & \frac{5\pi}{8} & \frac{3\pi}{4} & \frac{7\pi}{8} \\
0 & \frac{3\pi}{8} & \frac{3\pi}{4} & -\frac{7\pi}{8} & -\frac{\pi}{2} & -\frac{\pi}{8} & \frac{\pi}{4} & \frac{5\pi}{8} \\
0 & \frac{5\pi}{8} & -\frac{3\pi}{4} & -\frac{\pi}{8} & \frac{\pi}{2} & -\frac{7\pi}{8} & -\frac{\pi}{4} & \frac{3\pi}{8} \\
0 & \frac{7\pi}{8} & -\frac{\pi}{4} & \frac{5\pi}{8} & -\frac{\pi}{2} & \frac{3\pi}{8} & -\frac{3\pi}{4} & \frac{\pi}{8} \\
0 & -\frac{7\pi}{8} & \frac{\pi}{4} & -\frac{5\pi}{8} & \frac{\pi}{2} & -\frac{3\pi}{8} & \frac{3\pi}{4} & -\frac{\pi}{8} \\
0 & -\frac{5\pi}{8} & \frac{3\pi}{4} & \frac{\pi}{8} & -\frac{\pi}{2} & \frac{7\pi}{8} & \frac{\pi}{4} & -\frac{3\pi}{8} \\
0 & -\frac{3\pi}{8} & -\frac{3\pi}{4} & \frac{7\pi}{8} & \frac{\pi}{2} & \frac{\pi}{8} & -\frac{\pi}{4} & -\frac{5\pi}{8} \\
0 & -\frac{\pi}{8} & -\frac{\pi}{4} & -\frac{3\pi}{8} & -\frac{\pi}{2} & -\frac{5\pi}{8} & -\frac{3\pi}{4} & -\frac{7\pi}{8}
\end{bmatrix}
\end{array}
$$

Columns 1 & 5, 2 & 6, 3 & 7, and 4 & 8 have a $\pm\frac{\pi}{2}$ phase relationship on a term by term basis. These are combined using 3 dB couplers to form the circuit in Figure 19.

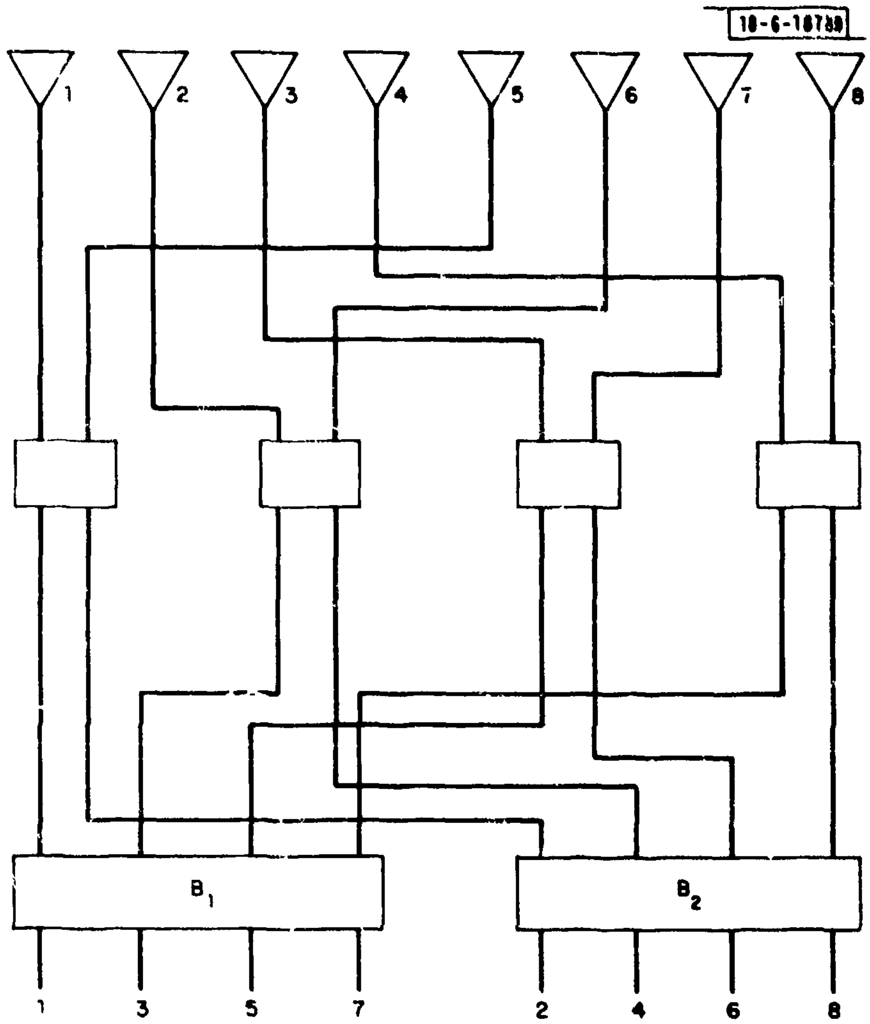


Fig. 19. Eight-element beam forming network - first reduction.

The B_1 and B_2 transfer matrices become:

$$B_1 = \frac{\sqrt{2}}{\sqrt{8}} e^{j \begin{bmatrix} 0 & \pi/4 & \pi/8 & 3\pi/8 \\ 0 & -(3\pi/4) & 5\pi/8 & -(\pi/8) \\ 0 & \pi/4 & -(7\pi/8) & -(5\pi/8) \\ 0 & -(3\pi/4) & -(3\pi/8) & 7\pi/8 \end{bmatrix}}$$

$$B_2 = \frac{\sqrt{2}}{\sqrt{8}} e^{j \begin{bmatrix} -(\pi/2) & \pi/4 & -(\pi/8) & 5\pi/8 \\ -(\pi/2) & -(3\pi/4) & 3\pi/8 & \pi/8 \\ -(\pi/2) & \pi/4 & 7\pi/8 & -(3\pi/8) \\ -(\pi/2) & -(3\pi/4) & -(5\pi/8) & -(7\pi/8) \end{bmatrix}}$$

Note now that adding $\pi/4$ to columns 2 and 4 of B_1 and to columns 1 and 3 of B_2 again produce a $\pm(\pi/2)$ relationship as before. Therefore using a coupler which results in a $\pm(\pi/2)$ relationship and subtracting $\pi/4$ produces the desired result. Figure 20 shows the resultant circuit.

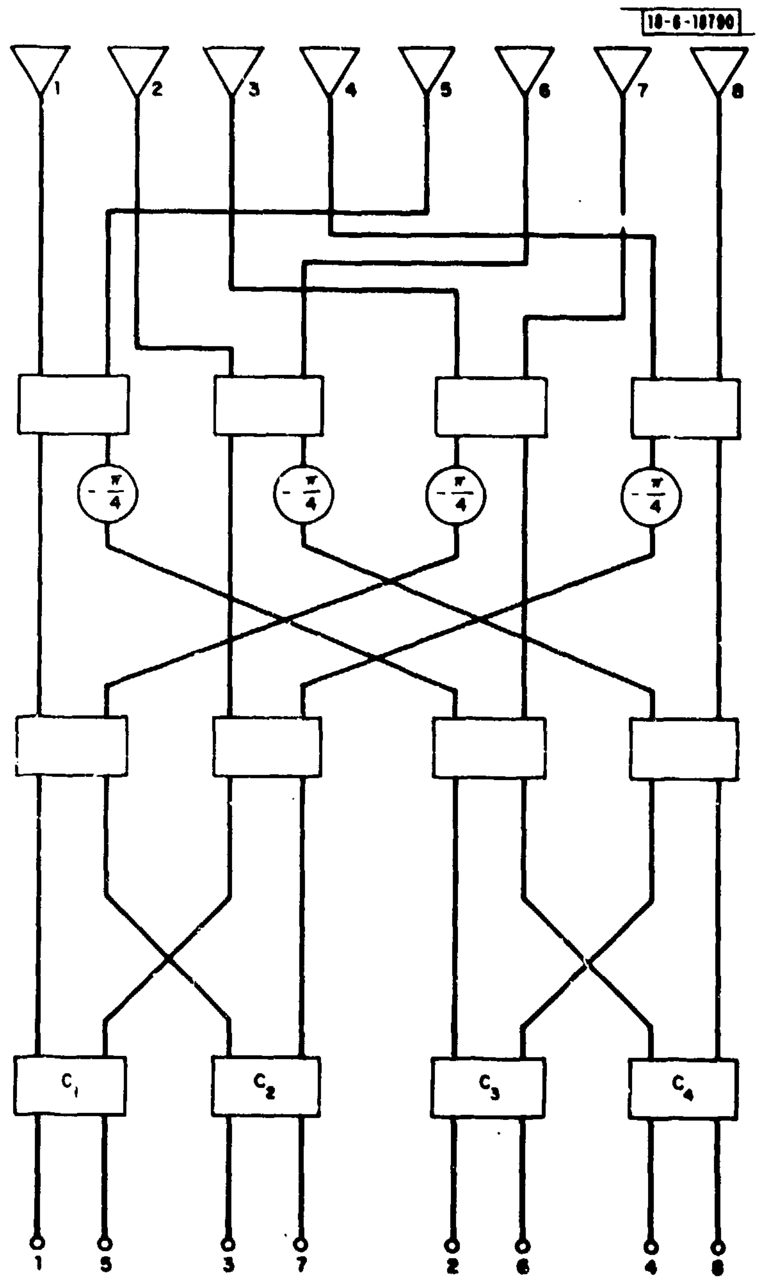


Fig. 20. Eight-element beam forming network - second reduction.

The C matrices are:

$$C_1 = (2/\sqrt{8})\, e^{j} \begin{bmatrix} 0 & \pi/8 \\ 0 & -(7\pi/8) \end{bmatrix}$$

$$C_2 = (2/\sqrt{8})\, e^{j} \begin{bmatrix} 0 & 5\pi/8 \\ 0 & -(3\pi/8) \end{bmatrix}$$

$$C_3 = (2/\sqrt{8})\, e^{j} \begin{bmatrix} -(\pi/4) & \pi/8 \\ -(\pi/4) & -(7\pi/8) \end{bmatrix}$$

$$C_4 = (2/\sqrt{8})\, e^{j} \begin{bmatrix} -(3\pi/4) & \pi/8 \\ -(3\pi/4) & -(7\pi/8) \end{bmatrix}$$

And note here that the addition of $3\pi/8$ to column 2 of C_1 and column 1 of C_4 results in a quadrature relationship. Also the addition of $\pi/8$ to column 1 of C_2 and column 2 of C_3 produces a quadrature relationship. Subracting these, as before, from the coupler outputs produces the circuit, neglecting terms which only affect the relative phase of the beams, shown in Figure 21. This circuit is identical to that shown in Figure 3 for the eight element Butler beam-forming network.

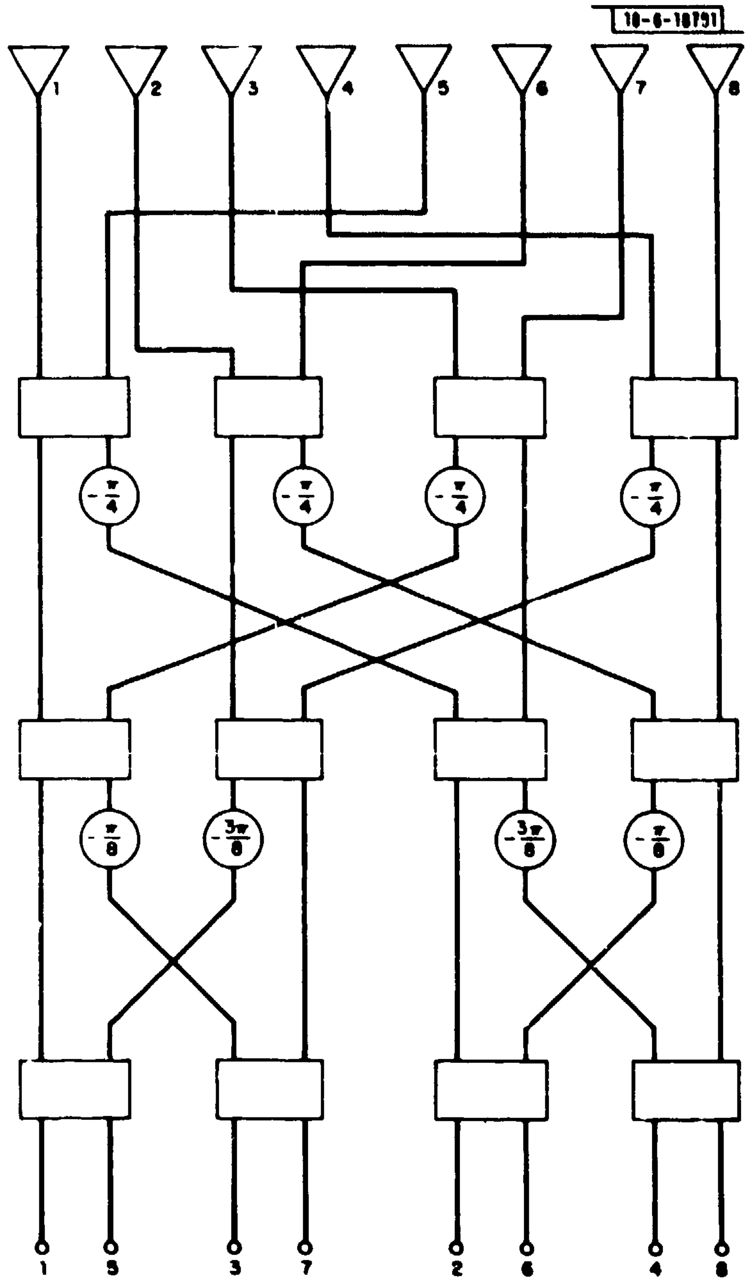


Fig. 21. Eight-element beam forming network - third reduction.

It should be noted that if, during the simplifying process, the required transfer matrices had an odd number of rows and columns, as would be the case for a 6 element network, the simplification procedure described for odd numbered arrays is applicable.

The result in the preceding example indicates that the Butler matrix is a special case of a more general beam forming network synthesis procedure. It is possible to apply the Nolan synthesis procedure directly without using the procedures described in this section and obtain the same result. This procedure, however, involves reordering the columns of the transfer matrix after each operation. The methods presented here for simplification of the networks involve less effort than does the direct application of the Nolan procedure.

VI. PLANAR ARRAYS

The phase taper for a linear array can be represented graphically as a line as in Figure 4. The phase taper for a planar array takes on an added dimension and can be represented as a plane. If the phase argument for an N element linear array is as in (5) (i.e., $(n-1)(k-1)\frac{2\pi}{N}$), then the phase taper will begin a new cycle at the N+1st element. Application of this property to planar arrays results in the synthesis of multibeam planar arrays described by Shelton.[7] First, the geometric configuration of the desired planar array is selected and a reference element (k=1) is arbitrarily chosen as in the seven element hexagon in Figure 22.

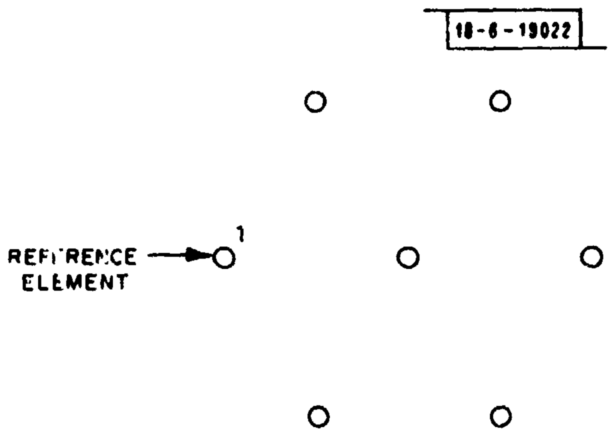


Fig. 22. Seven-element array.

Next, this array is inserted into a lattice structure of "phantom" arrays as in Figure 23.

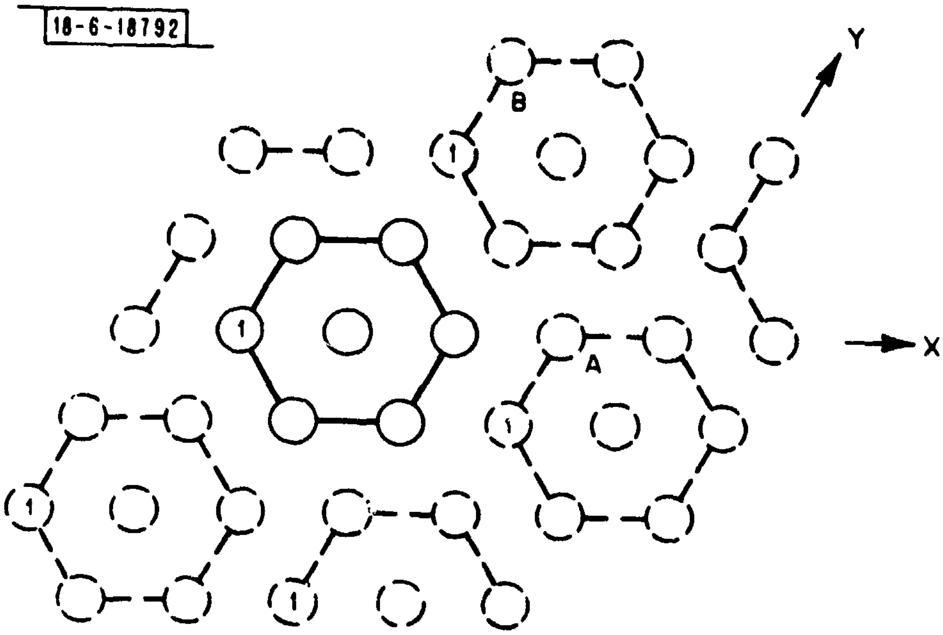


Fig. 23. Hexagonal lattice structure.

It can then be seen that the phase of the reference element of arrays A&B must differ from the reference element of the "real" array by $2\pi/n$ for the n^{th} beam. Since the phase taper is planar, one can write

$$m_x X + m_y Y = 2\pi(n-1) \tag{10}$$

where m_x and m_y are the slopes of the phase taper in the X and Y directions, respectively. If X and Y are measured in terms of the number of spaces between elements, then Equation (10) can be written as

$$K_x(n-1)\,\frac{2\pi}{N}\,X + K_y(n-1)\,\frac{2\pi}{N}\,Y = 2\pi(n-1) \tag{11}$$

where K_x and K_y are integers. The equation simplifies to

$$K_x X + K_y Y = N \tag{12}$$

and for the example of the 7-element array with the "phantom" arrays A and B, the two equations

$$3K_x - K_y = 7$$

$$K_x + 2K_y = 7$$

can be written for which

$$K_x = 3$$

$$K_y = 2.$$

This determines the phase increments for the 7-element array. These are tabulated for k=1 through 7 in Figure 24.

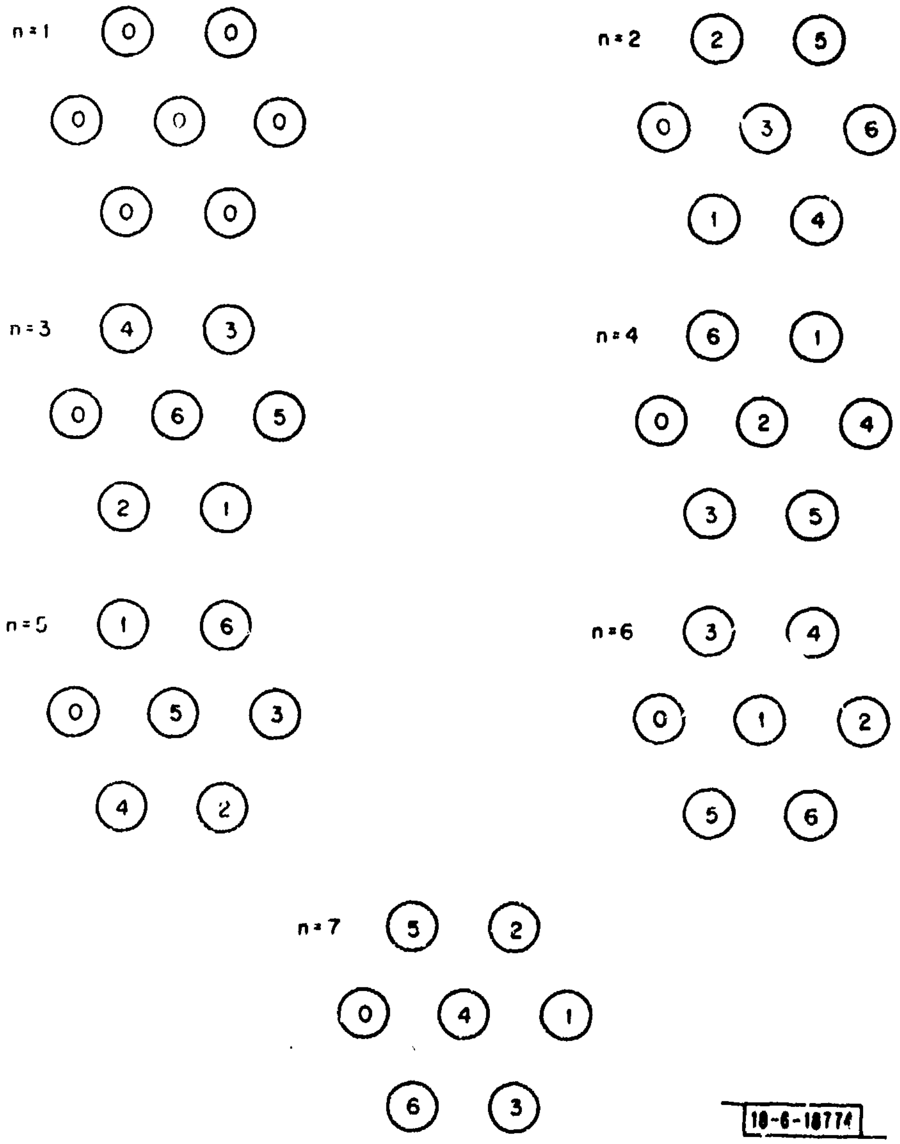


Fig. 24. Element Phases as a multiple of $\frac{2\pi}{7}$ for a 7-element hexagonal array.

Note that if these phase increments are tabulated with element numbers assigned as in Figure 25, the desired transfer matrix becomes:

$$A = (1/\sqrt{7})\, e^{j(2\pi/7)} \begin{array}{c} \left[\begin{array}{ccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 2 & 4 & 6 & 1 & 3 & 5 \\ 0 & 3 & 6 & 2 & 5 & 1 & 4 \\ 0 & 4 & 1 & 5 & 2 & 6 & 3 \\ 0 & 5 & 3 & 1 & 6 & 4 & 2 \\ 0 & 6 & 5 & 4 & 3 & 2 & 1 \end{array}\right. \end{array} \begin{array}{l} n=1 \\ n=2 \\ n=3 \\ n=4 \\ n=5 \\ n=6 \\ n=7 \end{array} \Bigg]$$

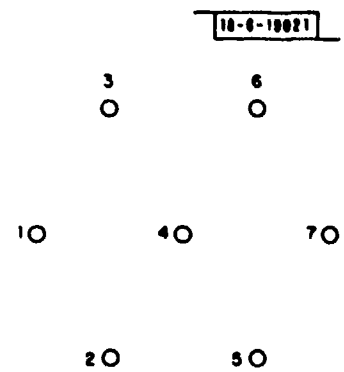


Fig. 25. Seven-element hexagonal array.

This matrix has transfer coefficients of the form

$$a_{n,k} = (1/\sqrt{7})\, e^{j(n-1)(k-1)\frac{2\pi}{N}}$$

which is no different from the matrix previously described for a linear array. The beam forming network for this type of array is identical to that used for a linear array.

Although this procedure allows the beam forming network to be applied to planar arrays, it is somewhat restricted. First, the array elements must be arranged such that they are on a regular lattice such as in the example; second, not all arrays even if configured in this way will result in use of the N beam-forming network. The 8-element array, for example, configured in a rectangular lattice reduces to two four element arrays by this method.

Random element spacing, subject to the restriction that the array elements lie on the lattice, is possible. This is an extension of the principle described for the Butler array. The appropriate elements in the "real" array are replaced by the elements in the "phantom" array.

VII. THE GENERAL ARRAY CASE

In the beginning of the discussion, the restrictions placed on the antenna array used with the beam forming network were that it was required to be a linear, equally spaced array of similar antenna elements of number equal to an integral power of 2. Subsequent discussion removed the equal spacing requirement and substituted the requirement that the element spacings must be integral multiples of some arbitarily chosen basic spacing interval. Section IV extends the array to allow any number of antenna elements by using the Nolan synthesis procedure. The restriction that the array be a linear array was removed in Section VI and the use of planar arrays configured on a regular lattice was discussed.

There are, inevitably, some design situations where more general types of arrays are required. For example, it may be necessary to configure the array in three dimensions rather than one or two; or, the array element patterns may be significantly different for one reason or another.

The Nolan synthesis is confined by only one restriction, that the specified network transfer function be a unitary matrix. This is another way of saying that the beam-forming network is lossless. Throughout the discussion thus far, only two unitary transfer matrices have been discussed, those of Equations (8) and (9). These do not exhaust the possibilities, however.

In the general case, a transfer matrix which is not unitary may be desired in order to achieve a particular set of array excitations. Such a network will be lossy. It is difficult to determine a minimum loss network which will have the desired transfer function; and if the network can be determined, it is very likely to be too lossy and to produce beams of unequal gain. Usually, however, there is some tolerance on the desired beam directions so that the desired transfer function can be considered to be approximate. Allowing some deviation from the desired transfer function, the problem then becomes one of finding a unitary transfer function which best approximates in some sense the desired transfer function.

One method of approximation is to minimize the mean square difference between the terms of the unitary matrix and the desired network transfer matrix. Using this as a basis for approximation, the unitary matrix which best approximates a given desired transfer function is given as

$$A = VT^T$$

where A is the resultant unitary matrix and V and T are eigenvector matrices associated with the product matrices SS^T and S^TS, respectively. S is the desired transfer matrix and $(\ )^T$ indicates the conjugate transpose of a matrix. The derivation of this expression is described in Appendix I. The matrix A will not necessarily represent a transfer function which will produce acceptable antenna beams. Much depends on the original desired transfer function S. If the desired beams are uniformly distributed in the region between grating lobe positions of the array, then A will provide a good approximation to the desired signal matrix. If, however, the desired beams are closely grouped with a relatively large angular separation between this group and the grating lobe positions, then the approximation will probably be a poor one. In more rigorous terms, the more completely the desired beam vectors span the N dimensional vector space of the NxN desired transfer matrix, the closer the approximation of A to S. This is simply a result of the fact that beam vectors of A will always completely span the vector space.

The antenna beams obtained with a beam-forming network A will usually be somewhat different in gain with respect to one another, the differences depending on the closeness of the approximation to the transfer function S. (This assumes that S specifies equal gain beams although this is not necessary. If S specifies unequal gains then the statement applies in a relative sense.) This results from the fact that the sidelobe structures of the individual beams will differ and that the beamwidths and shapes of the beams will also not be identical.

The approximation described coupled with the Nolan synthesis provides some significant advantages in most cases, however. Together, they represent a

straightforward procedure for the design of lossless beam-forming networks for arbitrary arrays. Since the desired transfer matrix S is completely arbitrary the array can take any conceivable configuration. The array element spacing can be anything. The array can be linear, planar, or three dimensional; it can consist of regularly spaced or randomly spaced elements. The array can be a filled array or a thinned array, and it can even be made up of elements with differing pattern gain characteristics. In addition, it is possible to specify aperture distributions to produce low sidelobes or to shape the beam. The degree to which the resultant design satisfies these requirements is, in general, dependent only upon how completely the specified signal vectors span the N dimensional vector space of the matrix.

The results of this approximation have been applied to the synthesis of beam-forming networks for randomly spaced thinned planar arrays of similar elements. The characteristics of two such arrays were computed. The first, a 7-element array with a single central element and 6 elements randomly spaced on the circumference of 10 foot radius circle. The desired beams were specified to be of equal gain and to be located on the vertices of a hexagon at an angular separation of 7° from the array boresight. A single beam was specified to be on boresight. Figures 26 and 27 show the computed 1 dB and 3 dB contours for the beams formed at 350 MHz using the transfer function A.

The second array computed was a 19-element array with 3 elements randomly located on a 15 foot diameter circle, 6 elements on a 30 foot diameter circle, and 10 elements on a 45 foot diameter circle. The desired beam directions were again chosen in a hexagonal pattern to form two concentric hexagons with a beam on the array boresight. Figures 28 and 29 show the computed 1 dB contours at 350 MHz for this array with a beam-forming network with the unitary transfer function A and the array configuration, respectively.

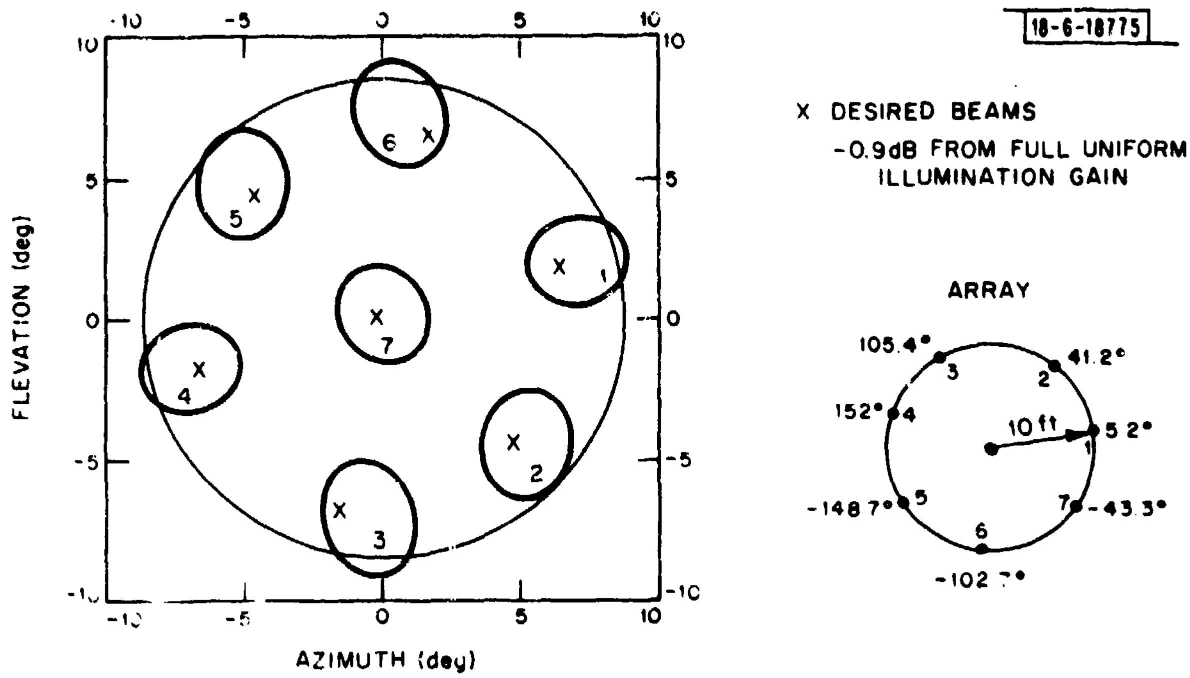


Fig. 26. Random circular array 1 dB contours.

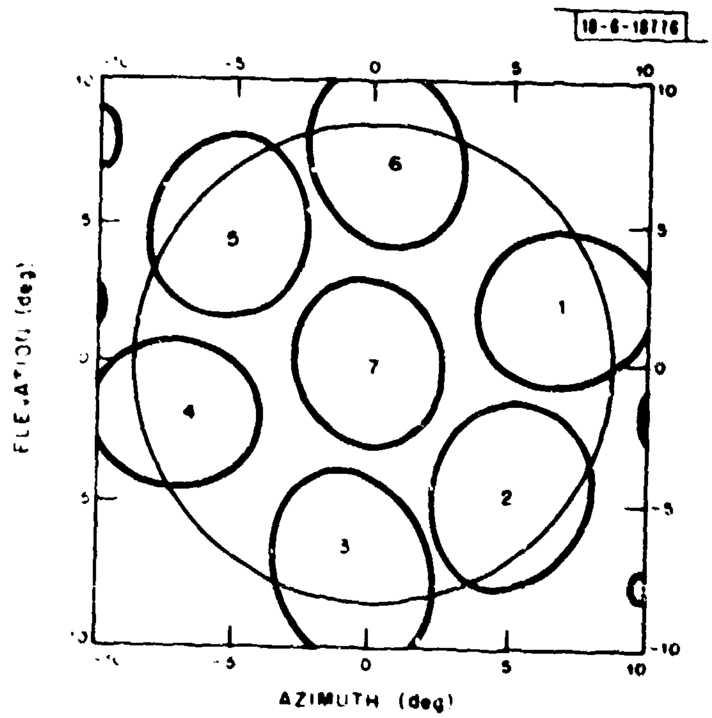
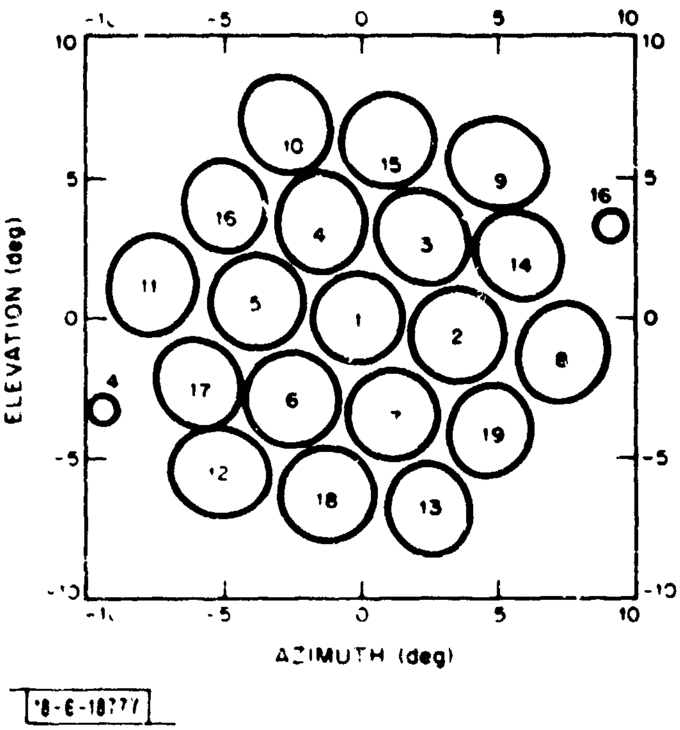


Fig. 27. 3 dB contours circular array same as preceding.



BEAM	GAIN (Array Factor)
1	12 6 dB
2	12 3
3	12 3
4	12 3
5	12 3
6	12 3
7	12 3
8	11 4
9	11 5
10	12 [illegible]
11	11 4
12	11 5
13	12 1
14	12 0
15	11 8
16	12 5
17	12 0
18	11 8
19	12 1

UNIFORM ILLUMINATION = 12 8 dB

Fig. 28. 19-beam array.

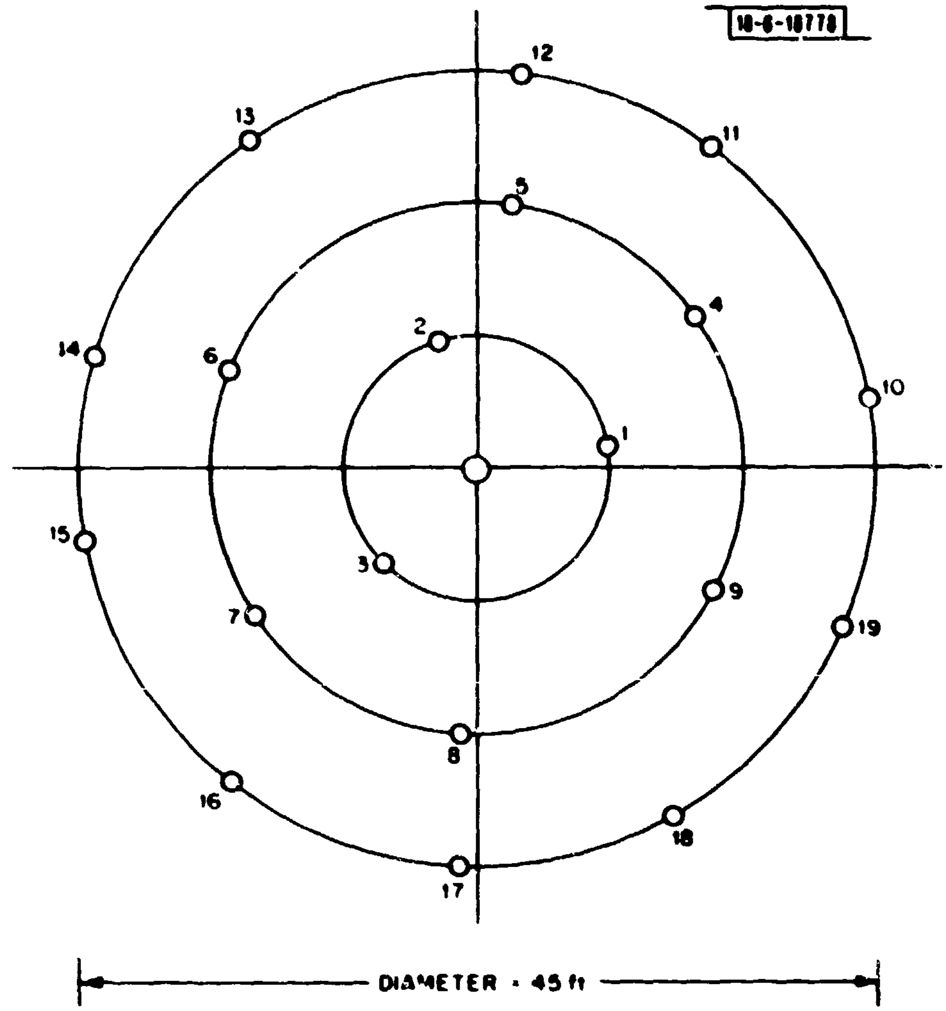


Fig. 9. 19-element random array.

VIII. SUMMARY

This discussion has attempted to describe possible methods of deriving lossless beam-forming networks for various types of arrays beginning with the well-known Butler array and extending this array to unequally spaced linear arrays, planar arrays and finally to the completely general type of array. For this last, it is indicated that it is not always possible to synthesize a lossless BFN but that there exists a least mean square optimization which leads to the synthesis of a lossless BFN which provides performance which approximates the desired performance.

APPENDIX I

Denoting the optimum unitary transfer function by A then as in Equation (4)

$$A^T A = I$$

and denoting the desired transfer function by S and premultiplying by A^T results in

$$A^T S = B \tag{A-1}$$

where B is approximately equal to I. If S is represented as the sum of the matrix A and some matrix R such that

$$A + R = S \tag{A-2}$$

then

$$A^T S = A^T(A + R) = I + A^T R = B$$

and

$$A^T R = B - I$$

which indicates that if A is a non trival (not all zeros) matrix, that it is desirable to minimize R in order to make B approximate I as closely as possible.

One way of minimizing R is to minimize the sum of the squares of the magnitudes of each term in R. Or,

$$\sum_{ij} |r_{ij}|^2 = \text{minimum}$$

or, with more convenient notation

$$||R||^2 = \text{minimum} \tag{A-3}$$

In order to determine the optimum transfer matrix A which will minimize R, it is necessary to assert two matrix transformations.[8] First, the eigenvalue transformation

$$U_1^T MM^T U_1 = \Lambda \tag{A-4}$$

where M is any matrix with n rows and p columns and U_1 is an NxN unitary matrix. The matrix Λ is diagonal with each term on the diagonal a real number called an eigenvalue of the matrix product MM^T. The columns of U_1 are called the eigenvectors of the matrix MM^T. If the matrix MM^T is known, then the eigenvalues Λ_i and the eigenvectors of U_1 can then be determined by straightforward (but tedious) computational methods. Second, using the notation of (A-3)

$$||U_2 N||^2 = ||N\, U_3||^2 = ||N||^2 \tag{A-5}$$

where U_2 and U_3 are any unitary NxN matrices and N is any NxN matrix. This can be demonstrated by carrying out the summation on a term by term basis and using the properties of the unitary matrix.

Referring back to Equation (A-3), it is desired to minimize $||R||^2$ and hence from Equation (A-2)

$$||S-A||^2 = \text{minimum} \tag{A-6}$$

There is a matrix transformation which will diagonalize the N by N matrix S such that

$$V^T S T = \Sigma \tag{A-7}$$

where V and T are unitary matrices and Σ is a diagonal matrix with positive real terms on the diagonal. This is shown by premultiplying (A-7) by V and postmultiplying by T^T and

$$VV^T\, STT^T = V\Sigma T^T$$

and since

$$VV^T = TT^T = I$$

$$S = V\Sigma T^T \tag{A-8}$$

From (A-8), $S^T = T\Sigma V^T$, since Σ is diagonal positive real and

$$\Sigma^T = \Sigma$$

then

$$SS^T = V\,\Sigma\, T^T T\, \Sigma^T\, V^T$$

$$= V\,\Sigma^2\, V^T$$

and

$$V^T SS^T V = \Sigma^2 \quad .$$

Since the matrix Σ^2 is diagonal positive real, then from Equation (A-4), V is the eigenvector matrix of SS^T and the terms of the matrix Σ, σ_i, are the square roots of the eigenvalues λ_i. Similarly,

$$
\begin{aligned}
S^T S &= T\,\Sigma\,V^T\,V\,\Sigma\,T^T \\
&= T\,\Sigma^2\,T^T
\end{aligned}
$$

and

$$T^T S^T S T = \Sigma^2$$

where it is seen that T is the eigenvector matrix of $S^T S$.

Using Eq. (A5) to re-write Equation (A-6) provides

$$||S-A||^2 = ||V^T S T - V^T A T||^2 = \text{minimum}$$

or

$$||\Sigma - V^T A T||^2 = \text{minimum} \quad .$$

The problem then is to minimize $||\Sigma-P||^2$ where Σ is diagonal positive real and P is the matrix $V^T AT$.

Writing as a term by term summation

$$||\Sigma-P||^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} |\Sigma-P|_{ij}^2$$

but since Σ is diagonal

$$||\Sigma-P||^2 = \sum_{\substack{i=1 \\ i\neq j}}^{N} \sum_{j=1}^{N} |p_{ij}|^2 + \sum_{i=1}^{N} |\sigma_i - p_{ii}|^2$$

and expanding

$$\sum_{\substack{i=1 \\ i \neq j}}^{N} \sum_{j=1}^{N} |p_{ij}{}^{2}| + \sum_{i=1}^{N} (|p_{ii}|^{2} - \sigma_{i}(p_{ii} + p^{*}{}_{ii}) + \sigma_{i}{}^{2})$$

where $(\ )^{*}$ indicates the conjugate.

And rewriting

$$||\Sigma - P||^{2} = \sum_{i=1}^{N} \sum_{j=1}^{N} |p_{ij}|^{2} + \sum_{i=1}^{N} [\sigma_{i}{}^{2} - \sigma_{i}(p_{ii} + p^{*}{}_{ii})] \qquad \text{(A-9)}$$

Now the double summation for the first term is positive and also $\sum_{i=1}^{N} \sigma_{i}{}^{2}$ is some positive number which is dependent upon how the original desired transfer matrix was determined. Since $||\Sigma - P||^{2}$ must be greater than or equal to zero, it is minimized when the negative term of Equation (A-9) is maximized, i.e.,

$$\sum_{i=1}^{N} \sigma_{i}(p_{ii} + p^{*}{}_{ii}) = \text{maximum} \qquad \text{(A-10)}$$

Now the matrix P is unitary since

$$P^{T}P = T^{T}A^{T}VV^{T}AT = T^{T}A^{T}AT = T^{T}T = I$$

and the maximum possible value of Equation (A-10) is achieved when $p_{ii}=1+j0$. If this is true, then the matrix P is the identity matrix I and

$$V^{T}AT = I$$

from which is follows that the optimum unitary beam forming network transfer function is

$$A = VT^T$$

where V and T are the eigenvector matrices associated with the product matrices SS^T and S^TS of the desired non-unitary transfer function S.

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Lossless beam forming networks using phase shifters and quadrature couplers are described. A technique for using the Butler network with randomly spaced arrays is discussed. A synthesis procedure for networks with N-element arrays is described. A method of using these networks with planar arrays is discussed. A least mean square algorithm for the determination of a beam forming network for an arbitrary array and arbitrary beam direction is developed. Computer pattern data for two thinned random planar arrays is included.